V FESTIVAL INTERNACIONAL DE MATEMÁTICA
De costa a costa
Matemática como lenguaje para interpretar nuestro entorno
29 al 31 de marzo, 2006

USING RATIOS AND PROPORTIONS TO UNDERSTAND THE SIZE AND SCALE OF THE SOLAR SYSTEM

Lori Lambertson [1]

Description:
Explore our solar system from a mathematical point of view: through ratios and proportions. We will use simple materials to create scale models of the earth, moon, sun, and of the other planets in our solar system. Learn more about the Festival's Coast to Coast model of the Solar System.

Activities:

Earth and Moon
Hole Punch Earth
Scale Model of the Solar System

I. Earth and Moon

Materials:
spheres of many sizes, at least one per student
meter stick or tape

To Do and Notice
1. Choose a sphere.
2. Assume that your sphere can be **either** the earth **or** the moon, and find a partner whose sphere could represent the earth to your moon, or could represent the moon to your earth. Trading spheres is fine. Try to come up with an earth and moon combination that you think is the correct ratio of the actual sizes of these planets.
3. After the correct ratio is revealed, trade as necessary so that you and your partner have the correct scale to model the earth and the moon.
4. Move away from your partner so that the earth and moon are what you think is the correct distance from each other on this scale.
5. After the correct ratio is revealed, move so that you and your partner are the correct distance apart from each other.
6. Moon partners: hold the "moon" at eye level without extending your arm.
7. Earth partners: determine what part of your hand just covers the "moon" when your arm is extended. What part of your hand is it?
8. Trade spheres with your partner and repeat steps 7.

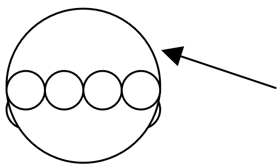
[1] The Exploratorium, San Francisco, California, EE. UU. e-mail: loril@exploratorium.edu

9. Given either the distance to the moon or its diameter, you can measure the other indirectly.

Note: Be sure to emphasize that only the earth's *diameter* is 4 times larger than that of the moon. Students should not assume that the earth is 4 times *bigger* than the moon, unless they can explain what they mean. The earth's surface area and volume are NOT 4 times the surface area and volume of the moon!

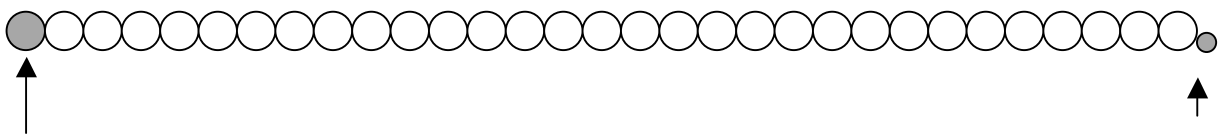
The correct ratios:

Four moon diameters will fit across one earth diameter. Assume the moon's diameter is about 2000 miles, then the earth's diameter is about 8000 miles.



earth

The earth and the moon are 30 earth diameters apart. 13,000 km (earth's diameter) times 30 = 390,000 km, the approximate distance between the earth and moon.



Earth **moon**

The Right Answers:

Planet	**Diameter** (actual)	**Diameter** (approx.)	**Distance** (actual)	**Distance** (approx.)
			to...earth	**to...earth**
moon	2160 mi 3476 km	2000 mi 3250 km 1/4 earth diameter	238,857 mi 384,404 km	240,000 mi 390,000 km 30 earth diameters
			to....sun	**to...sun**
earth	7918 mi	8,000 mi	93,000,000 mi	100,000,000 mi
	12,756 km	13,000 km	149,598,000 km	150,000,000 km

	4 moon diameters	100 sun diameters

---

sun	864,400 mi 1,392,000 km	1,000,000 mi 1,500,000 km 100 earth diameters - (consider earth diameter to be 10,000 miles or 15,000 km)

II. Hole Punch Earth

Create a scale model of the earth and sun using a hole punch as the Earth! Thanks to Coral Clark, who shared this idea with me.

Materials: (per pair)

- a hole punch circle (what you normally discard when you use a hole punch)
- metric ruler or meter stick
- large square of butcher paper (about 1 meter x 1 meter)
- 1 pair of scissors
- about 50 cm string
- 2 pencils
- calculator

To Do and Notice:

Provide students with the following information:

Earth's Diameter:	**12,756 km**	**~13,000 km**
Sun's Diameter:	**1,392,000 km**	**~1,300,000 km**
Earth - Sun Distance	**149,600,000 km**	**~150,000,000 km**

Give each pair of students a hole punch circle, explaining that it represents the planet earth. Students measure the diameter of the circle. Students find the correct scaling factor or set up a proportion to determine the diameter of the sun.

Scale Factor Method: For example, if the hole punch has a diameter of 1 cm, the following would be a way to find the scaling factor:

$$\frac{1\text{ cm}}{12{,}756\text{ km}} = \frac{1\text{ cm}}{12{,}756\text{ km}} \bullet \frac{1\text{ km}}{100{,}000\text{ cm}} = \frac{1}{1{,}275{,}600{,}000}$$

This scaling factor (which has no units) can be multiplied by the actual diameter of the sun to find the size of the scaled down version.

Proportion Method: To set this up as a proportion, we can compare the two size/distance ratios and solve for the unknown diameter of the scaled down sun, *x*. Simplify the ratio of the two diameters as shown.

$$\frac{1 \text{ cm } (\textit{scaled dia. of earth})}{x \text{ cm } (\textit{scaled dia. of sun})} = \frac{12{,}756 \text{ km } (\textit{diameter of earth})}{1{,}392{,}000 \text{ km } (\textit{diameter of sun})}$$

$$\frac{1 \text{ cm } (\textit{scaled dia. of earth})}{x \text{ cm } (\textit{scaled dia. of sun})} = \frac{\sim 13{,}000 \ (\textit{diameter of earth})}{\sim 1{,}300{,}000 \text{ km } (\textit{diameter of sun})}$$

Once the students have found the scaled down sun's diameter, they can draw it on the butcher paper. Show them how to use two pencils and string as a giant compass. Remind students of the definitions of radius and diameter. Students cut out their suns, which should be about 100 times the diameter of their hole punch earth. If you have the time and space (large outdoor area), have students then determine the correct scaled distance between the earth and sun. After they have made their calculations, they can measure or pace the distance outside.

What's Going On?

The sun is much larger and farther away than most people realize. The sun is about 100 earth diameters across and about 100 sun diameters away from earth. If the hole punch "earth" is .7 cm in diameter, then the correctly scaled sun should be about 70 cm in diameter. The correct distance for this particular earth/sun model would be about 70 meters!

What about volume?

If the earth/sun diameter ratio is 1 to 100, then the volume ratio must be cubed, or 1 to 1,000,000

III. Scale Model of the Solar System

Materials:

1 cm diameter marble to represent the sun
9 small pieces of paper for planet markers
9 sticks for the planet markers
clay or other material to support the sticks
calculators

To Do and Notice:

Choose a scale for your model. We will let the sun's diameter (1,392,000 km) equal to the diameter of a 1 cm marble.

Use the information below to find the correct scaled distances of each of the nine planets.

Proportion Method for Distance to Mercury Using Planet Distances and Scientific Notation:

scaled distance to planet = 5.79×10^7 (*actual distance to Mercury in km)*
1×10^{-2} m *(scaled dia. of sun)* 1.392 x 106 km *(dia. of sun)*

scaled distance to planet = 4.16×10^1
1×10^{-2} m

scaled distance to planet • 1×10^{-2} m = $4.16 \times 10^1 \bullet 1 \times 10^{-2}$ m

1×10^{-2} m

= 4.16×10^{-1} m = 41.6 cm

OR
Let the earth-sun distance equal 1 m.

Proportion Method for Distance to Mercury Using AU:

scaled distance to planet = .39 (*actual distance to Mercury in AU)*
1 m 1 AU

scaled distance to planet = .39 m = .39 cm
Use the following data below and the one of the proportion methods above to find the scaled distances to each of the planets.

The Planets (ingles)	**Los Planetas (español)**	**Mean Distance From Sun (km)** **Distancia al Sol (km)**	**Mean Distance From Sun (millions of km)** **Distancia al Sol (millones de km)**	**Distance in AU** **Distancia en UA**
Mercury		5.79×10^7	57.9	.39
Venus	**Venus**	1.082×10^8	108	.72

Earth	**La Tierra**	1.496×10^8	150	1
Mars	**Marte**	2.279×10^8	228	1.52
Jupiter	***Jupiter***	7.783×10^8	778	5.19
Saturn	**Saturno**	1.426×10^9	1,430	9.53
Uranus	**Urano**	2.871×10^9	2,870	19.13
Neptune	**Neptuno**	4.497×10^9	4,500	30
Pluto	**Pluton**	5.914×10^9	5,900	39.3
2003 UB313		1.455×10^{10}	14,550	97

Because of the large variation in planet sizes and distances, it is difficult to model both features at the same time. If you have time, model the distances in one lesson, and model the sizes in a different lesson.

The Planets (ingles)	**Los Planetas (español)**	**Mean Distance From Sun (km)** **Distancia al Sol (km)**	**Mean Distance From Sun (millions of km)** **Distancia al Sol (millones de km)**	**Equatorial Diameter (km)** **Diametro Ecuatorial (km)**
Mercury		5.79×10^7	57.9	4,880
Venus	**Venus**	1.082×10^8	108	12,100
Earth	**La Tierra**	1.496×10^8	150	12,800
Mars	**Marte**	2.279×10^8	228	6,790
Jupiter	***Jupiter***	7.783×10^8	778	143,000
Saturn	**Saturno**	1.426×10^9	1,430	120,000
Uranus	**Urano**	2.871×10^9	2,870	51,800
Neptune	**Neptuno**	4.497×10^9	4,500	49,500
Pluto	**Pluton**	5.914×10^9	5,900	3000(?)

Here is an on-line calculator of planetary distances and sizes:
http://www.exploratorium.edu/ronh/solar_system/

All of the planets, with the exception of Pluto, orbit the sun in essentially the same plane (called the ecliptic). The ecliptic plane then contains most of the objects orbiting the sun. This suggests that the formation process of the solar system resulted in a disk of material out of which formed the sun and the planets.

The 10th Planet

Last July, scientists at the California Institute of Technology discovered a new planet in the outer solar system. The planet, which hasn't been officially named yet, was found by Brown and colleagues using the Samuel Oschin Telescope at Palomar Observatory near San Diego. It is currently about 97 times farther from the sun than Earth, or 97 Astronomical Units (AU). For comparison, Pluto is about 40 AU from the sun. This places the new planet more or less in the Kuiper Belt, a dark realm beyond Pluto where thousands of small icy bodies orbit the sun. The planet appears to be typical of Kuiper Belt objects--only much bigger. Designated 2003 UB313, it is about 2,800 km across - a world of rock and ice and somewhat larger than Pluto.