Logarithms and Earthquake Magnitude

John Lahr 3/23/2006

Earthquakes are generated by slip on a fault and this slip can range from a very small displacement on a very small patch of a fault to a slip of 10's of meters over a fault with dimensions of 100's of kilometers! Seismometers can measure the amplitude of the ground vibrations generated by an earthquake. For example, at a distance of 100 km, the amplitude of ground motion for earthquakes too small to feel to the very largest recorded ranges from about

0.00000048 mm to 480 mm
(0.000000048 cm to 48 cm)
(0.00000000048 m to 0.48 m)

The thickness of a human hair is about .05 mm, so the range in ground motion is from 1/1000 of the thickness of a human hair to 10,000 times greater!

One way to deal with numbers that span a very large range is to use exponents of ten. In terms of exponents, the range of ground motion would be expressed:

4.8×10^{-7} mm to 4.8×10^{2} mm

This is clearly an improvement, but still a bit difficult to work with.

Logarithms provide a convenient way to deal with numbers that span a large range. In the case of earthquakes, the Richter magnitude scale has been defined as follows:

Magnitude = $\log_{10} (A / A_{zero})$, where A is the amplitude measured at 100 km distance and A_{zero} is 0.00000048 mm.

Magnitude	Ground amplitude (mm) at a distance of 100 km
0	0.00000048
1	0.0000048
2	0.000048
3	0.00048
4	0.0048
5	0.048
6	.48
7	4.8
8	48
9	480

How can an earthquake have a *negative* magnitude?

If the amplitude at 100 km is less than 4.8 x 10^{-7} then (A / A_{zero}) will be less than one, which yields a magnitude less than zero because the $\log_{10}$ of a number less than one will be negative. In practice earthquakes this small, although quite numerous, are usually too small to be recorded and located.

How is energy related to magnitude?

Seismologists have determined that the energy radiated by an earthquake is a function of both the amplitude of the waves and the duration of the earthquake. A very small earthquake is over in less than a second while for the largest events the fault may continue to slip for more than 5 minutes.

Considering the equation for magnitude (M):

$$M = \log_{10} (A / A_{zero})$$

$$A = A_{zero}\, 10^{M}$$

For each unit increase in M, the amplitude increases by a factor of 10.

Empirical studies have found that:

Energy is proportional to $10^{(1.5M)}$

Consider the energy (E1) from a magnitude M and from (E2) from magnitude M+1

$$E2/E1 = (10^{(1.5M + 1.5)})/(10^{1.5M})$$

$$E2/E1 = 10^{1.5} = 32$$

Thus, for each unit increase in magnitude, the energy increases by a factor of 32.

For two units of magnitude, the increase is a factor of 10^{3} or one thousand.

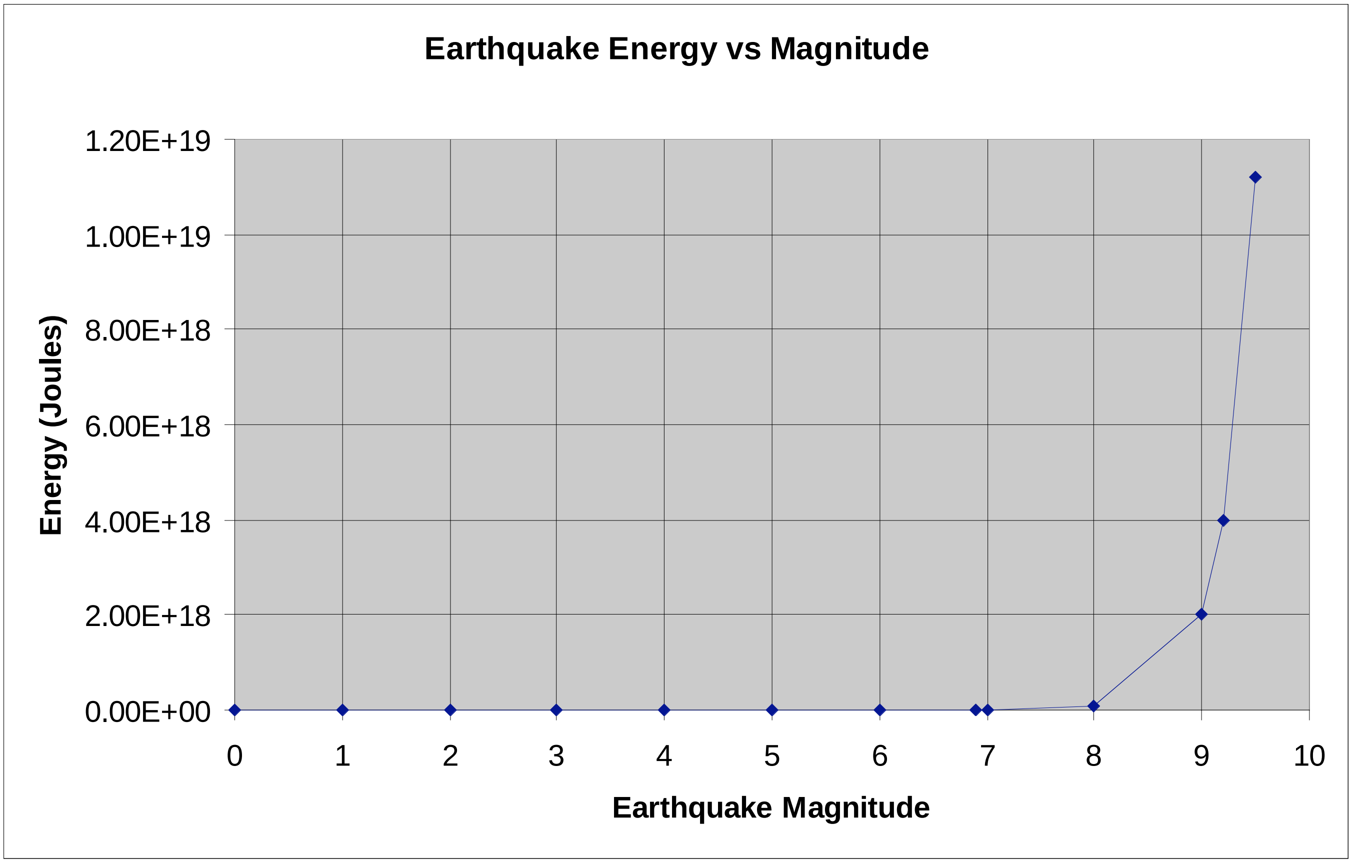
What are the actual energies involved?

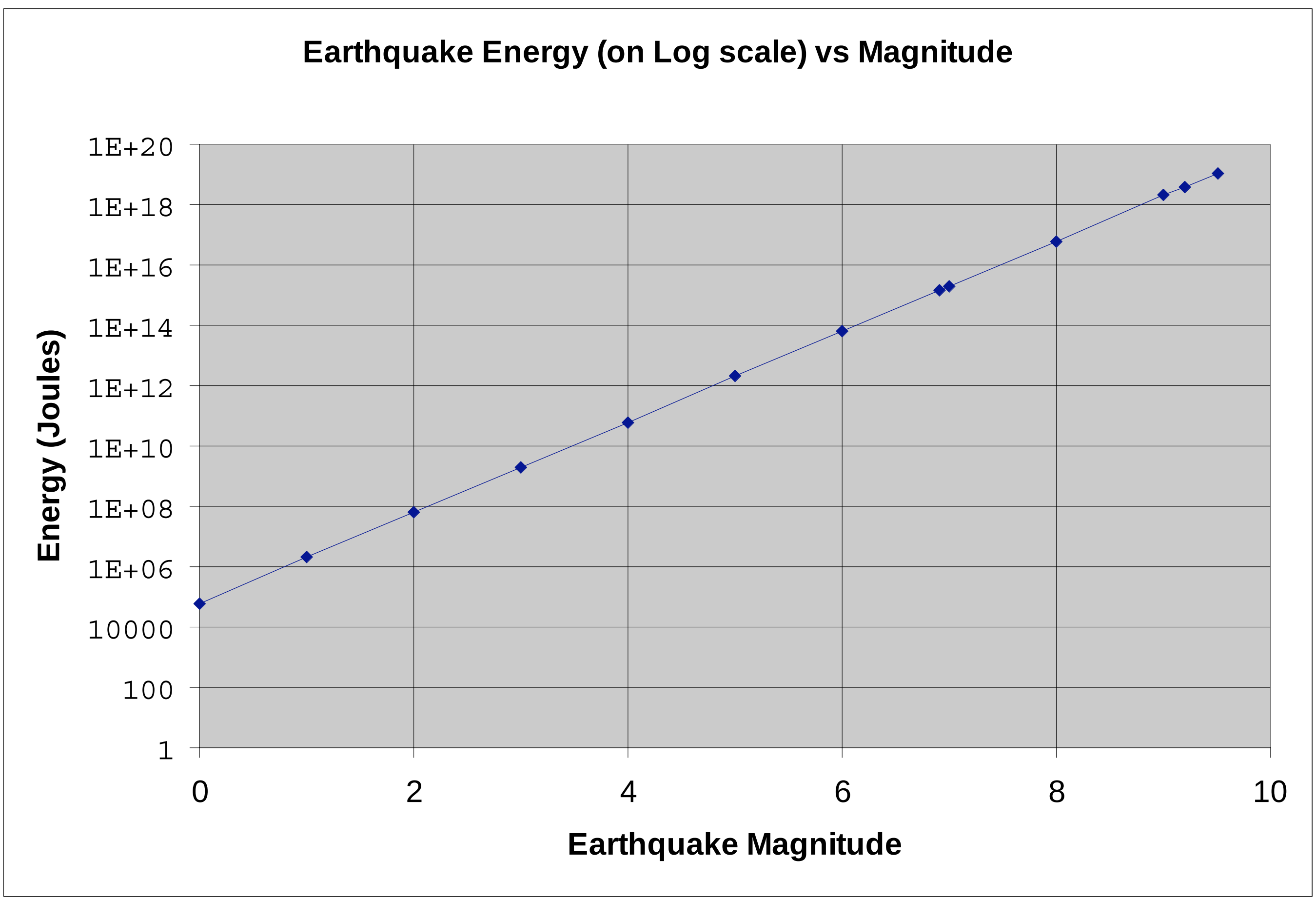
The equation relating energy (E) to magnitude is:

$$E = 10^{(1.5M + 4.8)} \text{ Joules}$$

Earthquake Energy as a Function of Magnitude				
Magnitude	Energy Equivalent Weight of TNT*		Energy in Joules	Notes
-3.0	0.001	ounces	0.200E+01	2.2 lbs dropped 8 in 1 kg dropped 20 cm
-2.0	0.032	ounces	0.631E+02	47 foot-pounds
-1.0	1.0	ounces	0.200E+04	150 lb person jumps down 10 ft 70 kg person jumps down 2.9 m
0.0	32	ounces	0.631E+05	
1.0	63	pounds	0.200E+07	
2.0	1	ton	0.631E+08	Only felt nearby.
3.0	32	tons	0.200E+10	Energy from 15 gallons of gasoline
4.0	1	kton	0.631E+11	Often felt up to 10's of miles away.
5.0	32	ktons	0.200E+13	Energy from 15,000 gallons of gasoline
6.0	1,000	ktons	0.631E+14	3.3 Hiroshima-size A bombs
6.9	22,500	ktons	0.141E+16	1995 Kobe, Japan, Earthquake
7.0	32,000	ktons	0.200E+16	0.47 mtons TNT
8.0	1,000	mtons	0.631E+17	15 mtons TNT
9.0	32,000	mtons	0.200E+19	430 x 10^9 kg of TNT; 470 mtons TNT
9.2	64,000	mtons	0.398E+19	1964 Alaska Earthquake - Second largest instrumentally recorded earthquake
9.5	180,000	mtons	0.112E+20	1960 Chile - Largest instrumentally recorded earthquake

The following graphs show the utility of using logarithms for plotting when the data span a large range of values.


Earthquake Energy vs Magnitude
Energy (Joules)
1.20E+19
1.00E+19
8.00E+18
6.00E+18
4.00E+18
2.00E+18
0.00E+00
0
1
2
3
4
5
6
7
8
9
10
Earthquake Magnitude


Earthquake Energy (on Log scale) vs Magnitude
Energy (Joules)
1E+20
1E+18
1E+16
1E+14
1E+12
1E+10
1E+08
1E+06
10000
100
1
0
2
4
6
8
10
Earthquake Magnitude

Stretching the News

Starting with a large page of newsprint, estimate how thick it is. Start by folding it in two as many times as possible and then using a ruler to measure the folded thickness. Divide this number by the number of layers in the folded stack.

Folds	Multiplier
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

Thickness of one sheet of newspaper: ________

Guess the thickness if the newspaper were placed on the first square of a chess board and then folded once for each additional square.

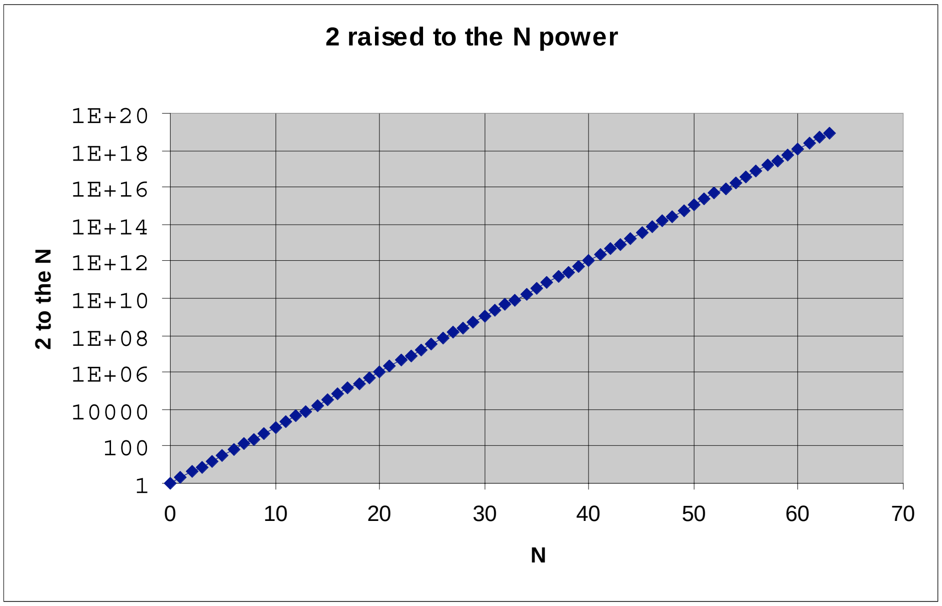
My guess is: ________________________

A chess board has 64 squares (8x8):

1	2	3	4	5	6	7	8
57	58	59	60	61	62	63	64

Compute the thickness, based on your estimate of the thickness of one sheet.
Computed thickness: _________________

My estimation:



Various distances	
Radius of the Earth:	6×10^3 km
Distance to the Sun:	1.5×10^8 km
Radius of the Solar System:	6×10^9 km
Distance to Nearest Star:	4×10^{13} km
Diameter of Our Galaxy:	9×10^{17} km

7 folds are about 1 cm thick, or 0.01 m thick, or 0.00001 km thick, or 10^{-5} km thick.
A single sheet is then $10^{-5}/2^7$ km thick

After 63 folds, the paper will be 2^{63} times thicker than a single sheet.
Or, $(10^{0.301})^{63}$ times thicker than a single sheet.
Or, about 10^{19} times thicker than a single sheet.

Total thickness = Thickness of a single sheet times 2^{63}
Total thickness = $(10^{-5}/2^7)(10^{63})$ km; Total thickness = $(10^{-5})(2^{56})$ km

$\text{Log}_{10}(2) = 0.301$, so $2 = 10^{0.301}$

Total thickness = $(10^{0.301})^{56}(10^{-5})$ km; Total thickness = $(10^{16.9})(10^{-5})$ km
Total thickness = $10^{11.9}$ km; Total thickness = $8*10^{11}$ km

The distance to the sun is about 150,000,000 km, or $1.5*10^8$ km

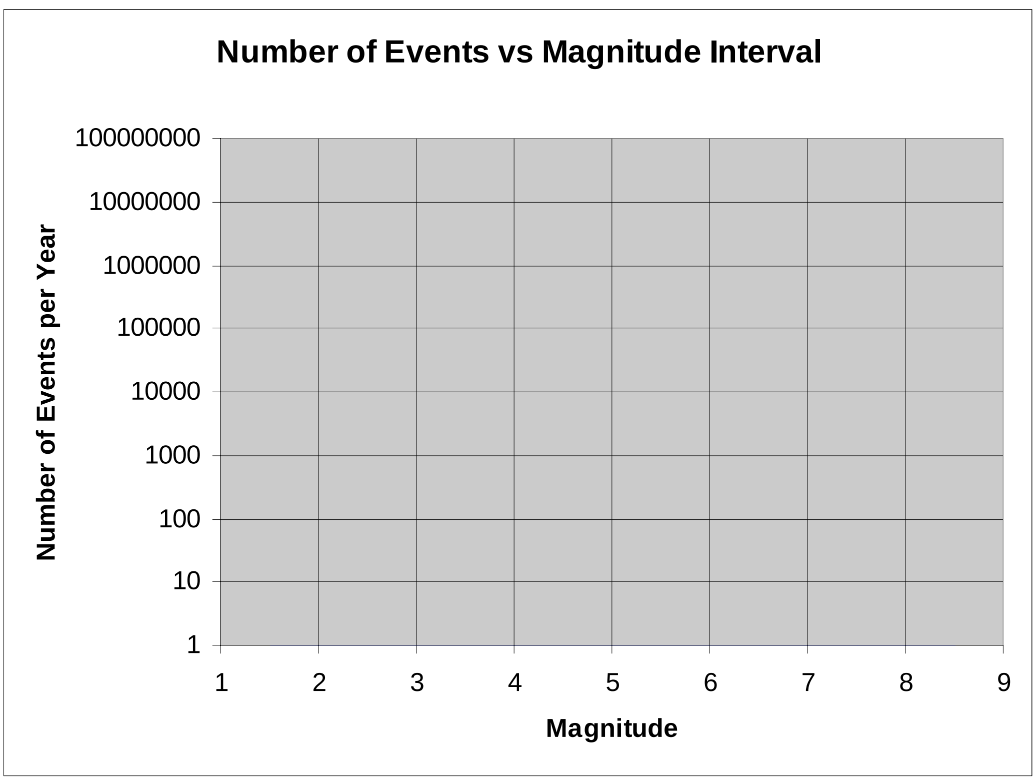
$8*10^{11} / 1.5*10^8 = 5.3*10^3$ (The paper is about 5,000 times thicker than the distance from the Earth to the Sun.
Thus, I've computed 63 folds of a piece of newspaper would reach to the Sun and back more than 2,500 times!

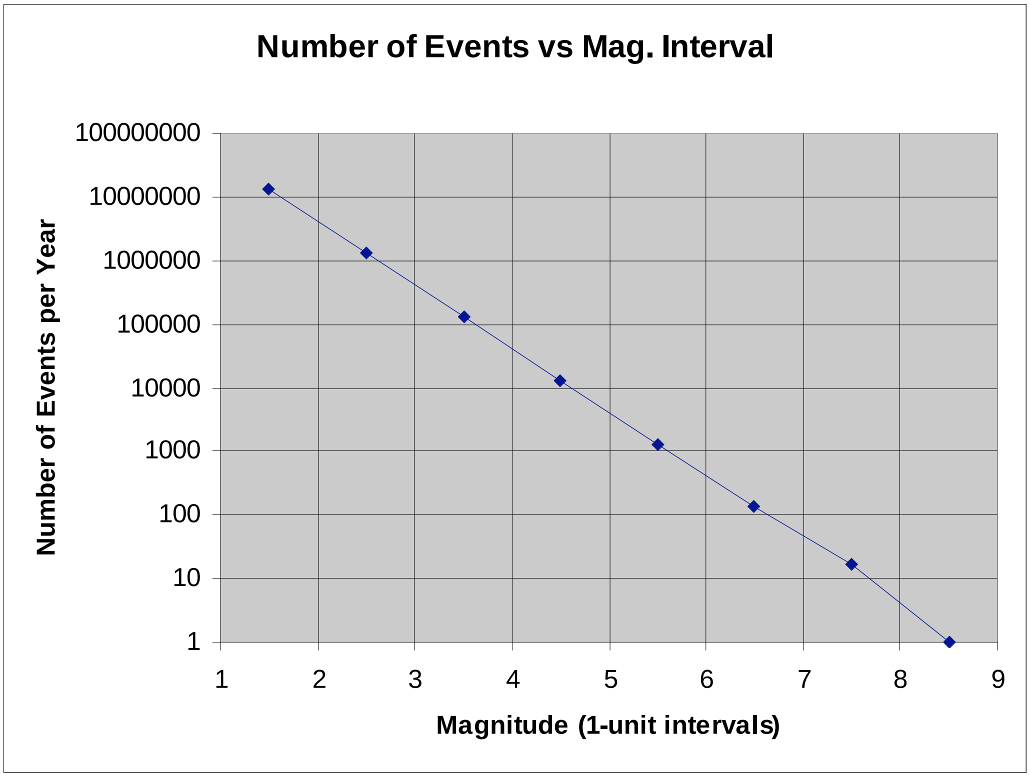
Earthquake frequency versus magnitude
Events with magnitudes above 6 (or smaller in areas of poor construction) can cause great damage, whereas events below magnitude 2 are rarely felt. Seismic instruments can detect events greater than 4.5 from anywhere in the world. The following describes the frequency of occurrence as a function of magnitude.

Descriptor	**Richter magnitudes**	**Global Average Frequency of Occurrence**
Micro	1.0-2.0	About 13,000,000 per year (estimated)
Very minor	2.0-2.9	About 1,300,000 per year (estimated)
Minor	3.0-3.9	130,000 per year (estimated)
Light	4.0-4.9	13,000 per year (estimated)
Moderate	5.0-5.9	1,319 per year
Strong	6.0-6.9	134 per year
Major	7.0-7.9	17 per year
Great	8.0 and greater	1 per year

Reference: http://neic.usgs.gov/neis/eqlists/eqstats.html

Plot these data on the graph below.





The slope is –1, so log(N) is proportional to –M.
N per year is proportional to 1/10**M
Recall the Energy (E) is proportional to 10**1.5M
Therefore, the total energy released per magnitude interval is proportional to 10**0.5M
There are fewer large earthquakes but the energy per event increases faster than their numbers drop!