# Mathematical Modeling and its Sociocritical Dimension

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**Abstract:** Among innovative teaching methodologies, it is important to highlight the use of social-critical dimension of mathematical modeling approach to solve problem situations that afflict contemporary society. Research related to mathematical modeling and its social-critical dimension has redefined its objectives, and is developing a sense of its own nature and potential of research methods in order to legitimize its pedagogical action. In this regard, it is necessary to discuss the importance of philosophical and theoretical perspectives found in social-critical dimensions of mathematical modeling and its epistemology as well. The importance of a learning environment that helps students to develop their social-critical efficacy is supported by the use of mathematical modeling.

**Keywords:** Mathematical Modeling, Social-critical Dimension, Social-critical Efficacy.

## Introduction

In order to reflect on the social-critical dimension of mathematical modeling, two questions are necessary:

- What is the role of schools in promoting students' social-critical efficacy?
- How do pedagogical practices currently used in the process of teaching and learning mathematics impact students' social-critical efficacy?

This allows us to determine the main goals for schools that relate to the development of creativity and criticality to help students apply different tools to solve problems faced in their daily lives as well as competencies, abilities, and skills to help them to live in society.

Unfortunately, in most cases, these goals are established in school curricula without the participation of community input. This curricular aspect contributes to an authoritarian education which unmotivates and promotes passivity in the teachers and students. The focus of education must be to prepare both teachers and students to be active, critical, and reflexive participants in society. However, in order to reach this objective, it is necessary that the community supports teaching and learning processes that help students to develop their social-critical efficacy. This means that teachers should be encouraged and supported to adopt pedagogical practices that allow their students to critically analyze problems that surround them in order to promote social justice.

## Conceptualizing Social-Critical Efficacy

One of the most important characteristics of teaching for social-critical efficacy is the emphasis on developing the students' critical analysis of the power structures of society. Another important feature of this kind of teaching is related to their students own reflection reflective abilities about social elements that underpin underlie life in a globalized world. Thus, critical perspectives in relation to social conditions affect their own experiences and may help them identify common problems and collectively develop strategies to solve them.

This is a form of transformatory learning based on students' previous experiences, which aims to create conditions that help to challenge predominant and harmful values. By using their own experiences combined with critical reflection on these experiences, students are able to develop their own rational discourse in order to create meanings that are necessary for the structural transformation of society (D'Ambrosio, 1990).

Rational discourse is a special form of dialogue in which all parties have the same rights and duties to claim and test the validity of their arguments in an environment free of social and political domination. In so doing, rational discourse provides an action plan that allows participants to dialogue, resolve conflicts, and engage collaboratively to solve problems in accordance to a set of specific rules. In this type of discourse, intellectual honesty, elimination of prejudices, and critical analysis of the facts are important aspects that allow dialogue to happen rationally (Rosa & Orey, 2007). This context is related to rational transformation that encourages the critical analysis of social phenomena. In this kind of educational environment, discourse, conscious work, intuition, creativity, criticality presents us with important elements that help students to develop their own social-critical abilities.

## Teaching for Social-Critical Efficacy

Education towards students' social-critical abilities places them at the center of the teaching and learning process. In this regard, classrooms become learning environments in which students to develop creative and critical abilities by applying transformatory pedagogical approaches. However, in order for this kind of education to be implemented in classrooms, it is necessary to discard many traditional pedagogical approaches (Jennings 1994). Teaching becomes a social-cultural activity that should induce students to the creation of knowledge instead of its transmission. This means that pedagogical transformation approaches are the antithesis of pedagogical approaches that seek to transform students into containers filled with information in a *banking mode* of education (Freire, 2000).

Currently, the debate between the two teaching approaches continues, but the discussions are centered in relation to content to be taught and limited in relation to the time required to teach these contents. In this regard, there is a need to elaborate a mathematics curriculum that promotes critical analysis, active participation, and social transformation (Rosa and Orey 2007). There is a need for curriculum changes that seek to prepare teachers and students to become critical and responsible citizens. This mission aims to use mathematics to find practical solutions to problems faced by society, which must be in accordance to the values and beliefs practiced by communities. This means that it is impossible to teach mathematics or other curricular subjects in a way that is neutral and insensitive to the true reality experienced by students (Fasheh, 1997).

Thus, an important objective for schools in a democratic society is to provide the necessary tools and information through relevant activities so that students have necessary tools to discuss and critically analyze curricular content by enabling them to solve daily problems and phenomena. In our point of view, mathematical modeling is a teaching methodology focused on students' social-critical efficacy because it engages them in relevant and contextualized activities, which allow them to be involved in the construction of mathematical knowledge.

## Theoretical Basis for the Social-Critical Dimension of Mathematical Modeling

The theoretical basis for the social-critical dimension of mathematical modeling has its foundations in *Sociocultural Theory* and the *Critical Theory of Knowledge*.

**Sociocultural Theory**

Learning occurs through socialization because knowledge is better constructed when students work in groups and act cooperatively in order to support and encourage each other. This approach allows students to reflect on complex problems embedded in real situations that help to construct knowledge by connecting it to other knowledge areas in an interdisciplinary way. According to this perspective, students' engagement with a sociocultural environment helps them to be involved in meaningful and complex activities. It is through social interaction (Vygotsky,1986) among teachers and students from distinct cultural groups that learning is initiated and established.

Thus, in the mathematical modeling process, the social environment also influences cognition in ways that are related to cultural context. In this context, collaborative work between groups of teachers and students makes learning more effective as it generates levels of mathematical thinking through the use of socially and culturally relevant activities Thus context allows the use of a *dialectical constructivism* because the source of knowledge is based on social interactions between students and environments in which cognition is the result of cultural artifacts in these interactions (Rosa & Orey, 2007).

**Critical Theory of Knowledge**

Studies of Habermas' *Critical Theory of Knowledge* reinforces the importance of social context for the teaching and learning processes because this theory promotes the development of students' critical consciousness so that they are able to analyze how social forces shape their lives. This analysis occurs through intellectual strategies such as interpersonal communication, dialogue, discourse, critical questionings, and proposition of problems taken from reality.

The effects of social structure influence distinct knowledge areas purchased by individuals in the social environment, and are partly determined by interests that stimulate and motivate these individuals. Thus, in this this theory it is recognized that there are three knowledge domains (Habermas,1971):

a) *Technical Knowledge (prediction)* is defined by the way individuals control and manipulate the environment. It is gained through empirical investigations and governed by technical rules. In the mathematical modeling process, students apply this instrumental action when they observe the attributes of specific phenomena, verify if a specific outcome can be produced and reproduced, and know how to use rules to select different and efficient variables to manipulate and elaborate mathematical models (Brown, 1984).

b) *Practical Knowledge (interpretation and understanding)* identifies individuals' social interaction through communication. In the mathematical modeling process, students communicate by using hermeneutics (written, verbal, and non-verbal communication) to verify if social actions and norms are modified by communication. It is in this kind of knowledge that meaning and interpretation of communicative patterns interact to construct and elaborate the community understanding that serves to outline the legal agreement for the social performance.

c) *Emancipatory Knowledge (criticism and liberation)* is defined by the acquisition of insights that seek to emancipate individuals from institutional forces that limit and control their lives. It is necessary to determine social conditions that cause misunderstandings in the communication process, tactics that may be used to release particular oppressive and repressive forces, and risks that are involved in these tactics. The objective of this kind of knowledge is to emancipate individuals from diverse modes of social domination. In the mathematical modeling process, insights gained through critical self-awareness of the elaboration of mathematical models are emancipatory in the sense that students may be able to recognize the correct reasons to solve problems faced by their communities. During this process, knowledge is gained by self-emancipation through reflection leading to a transformed consciousness.

However, learning begins to be generated in the technical knowledge in conjunction with the social existence through interactive and dialogical activities. In the mathematical modeling process, this approach helps students

to take ownership of the emancipatory knowledge. In this perspective, knowledge is translated in an interdisciplinary and dialogical ways so they can be used as instruments for social transformation.

**Determining an Epistemology of the Social-Critical Dimension of Mathematical Modeling**

Currently, there is no general consensus on specific epistemologies for social-critical dimensions of mathematical modeling, which we describe as a process that involves the elaboration, critical analysis, and validation of a model that represents a system taken from reality. In this regard, mathematical modeling can be considered as an artistic process because in the process of elaboration of a model, the modeler needs to possess mathematical knowledge as well as developing a sense of intuition and creativity that enables interpretation (Biembengut & Hein, 2000). In so doing, students need to work in a motivating learning environment so that they are able to develop and exercise their creativity and criticality through critical analysis of along with the generation and production of knowledge.

According to this context, mathematical modeling may be considered a learning environment in which students are invited to inquire and investigate problems that come from other areas of reality (Barbosa, 2001). In this learning environment, students work with real problems by using mathematics as a language for understanding, simplifying, and solving these situations in an interdisciplinary fashion (Bassanezi, 2002). Mathematical modeling is a method using applied mathematics that was transposed to the field of teaching and learning as one of the ways to use reality in the mathematics curriculum (Barbosa, 1999).

From this perspective, there are at least three distinct mathematical modeling pedagogical practices that may be used in the school curriculum (Barbosa, 2001):

a) *Case 1*: Teachers choose a problem, a situation, or a phenomenon and then describes it to the students. According to the curriculum content to be developed, teachers provide students with necessary mathematical tools that are suitable to the elaboration of the mathematical models in order to solve the proposed problem. In our opinion, this is the first step to integrate mathematical modeling to teaching and learning processes. However, for the development of social-critical efficacy, there is need for active involvement in the process of teaching and learning (Rosa & Orey, 2007).
b) *Case 2*: Teachers suggest and elaborate the initial problem. Students need to investigate the problem by collecting data, formulating hypotheses, and making necessary modifications in order to develop the model. Students themselves are responsible for conducting the activities proposed in order to develop the modeling process. One of the most important stages of the modeling process refers to the elaboration of a set of assumptions, aiming to simplify and solve the mathematical model to be developed. In order to work with activities based on the social-critical dimensions of modeling, it is necessary that students relate these activities to problems found in their community and/or reality (Rosa, Orey, & Reis, 2012). For example, such activity may be related to the concentration of pollutants in a river, which may help students to reflect on the mathematical aspects involved in this problem, enabling them to understand this phenomenon so they can critically solve this situation by turning it into positive action for their community.
c) *Case 3*: Teachers facilitate modeling processes by allowing students to choose their own themes that have particular value and interest. Then, students are encouraged to develop a project in which they are responsible for all stages of the process, that is, from formulation of the problem to the validation of the solution. The supervision by teachers is constant during the mediation of the modeling process, and enables students' social-critical engagement in the proposed activities.

However, even though there may be some disagreement regarding the use of a specific mathematical modeling pedagogical practices, it is possible to conduct activities, experiments, investigations, simulations, and research projects that interest and stimulate students at all educational levels. Thus, the choice of the approach to be used by teachers depends on the content involved, the maturity level of the students and the teachers' confidence with the modeling process in the classroom. However, we emphasize that the critical analysis of the results obtained in either approach must be highly encouraged and developed.

During the development of the modeling process, the problems chosen and suggested by teachers or those selected by students must be used to get them to critically reflect on all aspects involved in the situation to be modeled. These aspects are related to interdisciplinary connections, the use of technology, and the discussion of environmental, economic, political, and social issues. Thus, the use of content in the social-critical context is directed towards the critical analysis of problems faced by the community.

Reflective aspects are related to an *open* aspect or approach to the mathematics curriculum because its pedagogical practices offer activities that apply multiple perspectives, which require constant critical reflection on these solutions. However, the open nature of modeling activities may be difficult for students to initially understand and develop a model that satisfactorily represents the problem under study (Barbosa, 2001). Thus, the dialogical and mediator role of the teachers is very important during the modeling process.

## An Emancipatory Approach of the Social-Critical Dimension of Mathematical Modeling

The social-critical dimension of mathematical modeling may considered an extension of the Critical Theory of Knowledge. In this regard, the emancipatory approach directs the educational objectives by addressing social and political issues in the pedagogical practices used in educational systems.

According to the Brazilian National Curriculum for Mathematics (Brazil, 1998), students need to develop their own autonomous ability to solve problems, make decisions, work collaboratively, and communicate effectively. This approach is based on abilities, which help students to face challenges posed by society by turning them into flexible, adaptive, reflexive, critical, and creative citizens. This perspective is also related to the sociocultural dimensions of mathematics, which is closely associated with the ethnomathematics as program (D'Ambrosio, 1990). This aspect emphasizes the role of mathematics in society by highlighting the necessity to analyze the role of critical thinking about the nature of mathematical models as well as function of modeling to solve everyday challenges.

## The Process of the Social-Critical Dimension of Mathematical Modeling

Mathematical modeling provides concrete opportunities for students to discuss the role of mathematics as well as the nature of their models as they study systems taken from reality (Sriraman & Kaiser, 2006).

In accordance to this point of view, mathematical modeling may be understood as a language to study, understand, and comprehend problems faced community (Bassanezi, 2002). For example, mathematical modeling is used to analyze, simplify, and solve daily phenomena in order to predict results or modify the characteristics of these phenomena.

In this process, the purpose of mathematical modeling becomes the ability to develop critical skills that enable teachers and students to analyze and interpret data, to formulate and test hypotheses, and to develop and verify the effectiveness of mathematical models. In so doing, the reflections become a transforming action, seeking to reduce the degree of complexity through the choice of a system that can represent it (Rosa & Orey, 2007).

By developing strategies that encourage students to explain, understand, manage, analyze, and reflect on all parts of this system, the process optimizes pedagogical conditions for teaching and learning so that students understand a particular phenomenon in order to act effectively and transform phenomenon according to the needs the community. Figure 1 shows the social-critical mathematical modeling cycle.

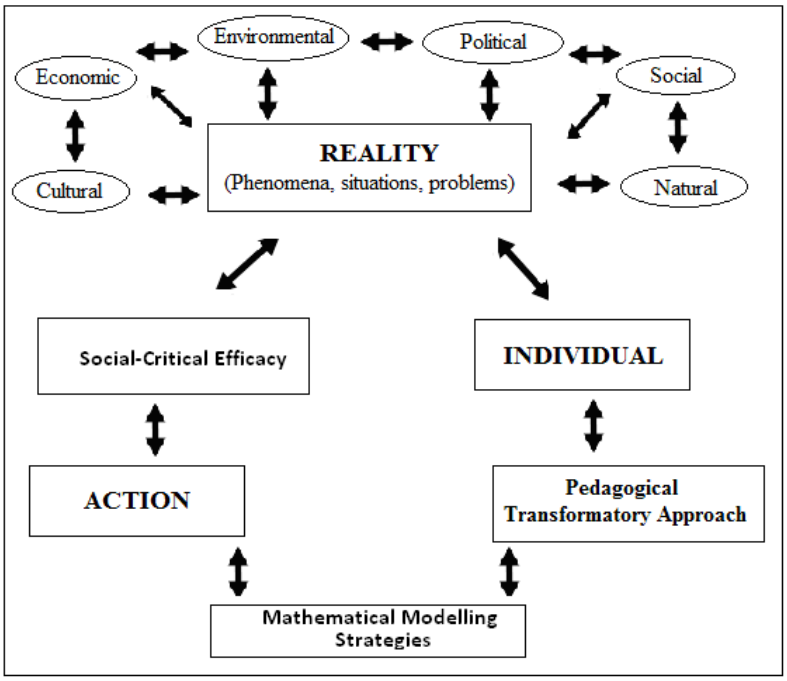


**Fig. 1:** Social-critical mathematical modeling cycle

The application of the social-critical dimension of modeling allows mathematics to be seen as a dynamic and humanized subject. This process fosters abstraction, the creation of new mathematical tools, and the formulation of new concepts and theories. Thus, an effective way to introduce students to mathematical modeling in order to lead them towards the understanding of its social-critical dimension is to expose them to a wide variety of problems or themes. As part of this process, questionings are used to explain or make predictions about the phenomena under study through the elaboration of models that represent these situations (Rosa & Orey, 2007).

However, the elaboration of mathematical models does not mean they will develop a set of variables that are qualitative representations or quantitative analysis of the system because models are understood as approximations of reality. Since mathematical modeling is a process that checks whether parameters are critically selected for solutions to models in accordance to their interrelationship to selected variables from holistic context of reality, it is not possible to explain, know, understand, manage, and cope with reality outside of a holistic context (D'Ambrosio, 1993). From a social-critical context, mathematical modeling is impossible to work without the theories and techniques that facilitate solutions for models that are not simply memorized and then forgotten. Traditional learning often prevents time for students to learn creativity, conceptual elaboration, and the development of logical and critical thinking.

Social-critical dimensions of modeling are based on autonomy, which aims to facilitate the expansion of world views, autonomous thinking, and contributions towards the full exercise of citizenship. According to this perspective, social-critical dimensions of mathematical modeling facilitate the competencies, skills, and abilities necessary for teachers and students to play a transformative role in society (Rosa & Orey, 2006).

## Final considerations

Fundamental characteristics of teaching towards social-critical efficacy emphasize a critical analysis of power structures of society through modeling. As well, modelers are encouraged to reflect on social elements that underpin our increasingly globalized world. Thus, a student's critical perspective in relation to social conditions

affect their own experiences help them to identify common problems and collectively develop strategies (D'Ambrosio, 1993).

This is unique and transformatory form of learning creates conditions that help teachers and students to work together to challenge worldviews and values dominant in our society. Through these experiences, students are guided towards to developing rational discourse by creating meanings that are necessary for the structural transformation of society (Freire 2000). This transformation involves critical analysis of social phenomena through the elaboration of data-based mathematical models.

In this context, mathematical modeling becomes a teaching methodology that focuses on the development of a social-critical efficacy that engages students in a contextualized teaching-learning process that allows them to get involved in the construction of solutions of social significance (Rosa & Orey, 2007).

This social-critical dimension of mathematical modeling is based on the comprehension and understanding of reality, in which students reflect, analyze and take action on this reality. When we borrow systems from reality, students begin to study the symbolic, systematic, analytical and critically contexts to their work. In this regard, starting from real problem situations, students learn to make hypotheses, test them, correct them, make transfers, generalize, analyze, complete and make decisions about the object under study. Thus, using social-critical mathematical modeling can explain ways to work with reality through a transformational action that reduces its complexity, and which allows students to explain, understand, manage and find solutions to their own interests and problems.

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